IDENTIFICATION OF HEAT-TRANSFER COEFFICIENTS IN A POROUS BODY FROM THE SOLUTION OF AN INVERSE PROBLEM

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An approach is proposed for determining the internal heat-transfer coefficient and effective thermal conductivity of a porous body. The approach is based on an iterative method of solving an inverse coefficient problem of heat conduction.

The determining parameters in calculating the thermal conditions of a porous body are the internal heat-transfer coefficients and the heat-transfer coefficient at the inlet to the body, the effective thermal conductivity of the porous frame, and the blowing rate, which is dependent on the pressure drop and the hydraulic resistance of the system.

The internal heat-transfer coefficients of porous bodies in steady-state regimes can be determined from the results of numerous heat experiments using, for example, the methods presented in [1-6].

The present work proposes an algorithm for identifying the internal heat-transfer coefficients and effective thermal conductivity of a porous frame from the solution of an inverse problem by means of an iterative method [7].

We will examine the following inverse coefficient problem of heat transfer for a porous body in the form of an infinite plate $0 \leqslant x \leqslant b$. It is necessary to use $(n-1)$ known nonsteady values of temperature inside the body and type I boundary conditions to determine the internal heat-transfer coefficient and effective thermal conductivity of the body. We prescribe the initial temperature distributions for the solid and gaseous phases, the law of change in coolant discharge over time, the hydraulic characteristics of the body, and the dependences of the remaining thermophysical characteristics of the material of the porous frame and the injected coolant on the corresponding temperature.

We write the problem as follows:

$$C_s \frac{\partial T_s}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_s \frac{\partial T_s}{\partial x}\right) - \alpha_V (T_s - T_g), \tag{1}$$

$$(\rho C_p)_g \frac{\partial T_g}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_g \frac{\partial T_g}{\partial x}\right) - \rho v C_{p_g} \frac{\partial T_g}{\partial x} + \alpha_V (T_s - T_g), \tag{2}$$

$$0 < x < b; \quad 0 < \tau \leqslant \tau_m,$$

$$T_s(x, 0) = \xi_s(x); \quad T_g(x, 0) = \xi_g(x), \tag{3}$$

$$T_s(0, \tau) = \varphi_s(\tau); \quad T_g(0, \tau) = \varphi_g(\tau), \tag{4}$$

$$T_s(b, \tau) = f_s(\tau); \quad T_g(b, \tau) = f_g(\tau), \tag{5}$$

$$T_s(x_i, \tau) = f_i(\tau), \quad i = 2, 3, \ldots, n. \tag{6}$$

The density of the gas is determined from the equation of state

$$\rho_g = \frac{pM_g}{8314 T_g}, \quad \text{kg/m}^3, \tag{7}$$

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where the pressure is calculated by integrating, over x, the differential equation describing the modified Darcy's law:

$$-\frac{dp}{dx} = \alpha\mu v + \beta\rho v^2. \tag{8}$$

We will solve the stated problem using an iterative algorithm to solve an inverse heat-conduction problem constructed by gradient methods. Recent investigations [7] showed that the algorithm is quite effective for solving imperfectly formulated problems.

We will transform imperfectly stated problem (1)-(6) into an extreme problem: find the functions $\alpha_V = \alpha_V(\rho v, \alpha\beta, ...)$ and $\lambda_{s\,\mathrm{eff}} = \lambda_s(T_s)$ from the condition of the minimum of the mean square error

$$J(\alpha_V, \lambda_s) = \sum_{i=2}^{n} \int_0^{\tau_m}]T_s(x_i, \tau) - f_i(\tau)]^2 \, d\tau \tag{9}$$

with limitations (1)-(3). We determine $T_s(x_i, \tau)$ from the solution of the direct heat-transfer problem in the porous body.

To calculate the gradient of functional (9), in the gradient method it is necessary to solve the inverse problem, using the solution of the conjugate problem [7].

We will write the conjugate problem for the given case. Regarding the stated problem (1)-(5) as a multilayered problem, we obtain a formula for the increment of functional (9) with a change in the sought quantities α_V and λ_s by small amounts $\Delta\alpha_V$ and $\Delta\lambda_s$, respectively. Here, T_s and T_g receive small increments in $z(x, \tau)$ and $u(x, \tau)$, satisfying the equations

$$\frac{\partial C_s z_i}{\partial \tau} = (\lambda_s z_i)'' + \alpha_V (u - z_i) - \left(\Delta\alpha_V + u\frac{\partial \alpha_V}{\partial T_g}\right)(T_{s_i} - T_g) + \frac{\partial}{\partial x}\left(\Delta\lambda_s \frac{\partial T_s}{\partial x}\right), \quad i = 1, 2, \ldots, n; \tag{10}$$

$$\frac{\partial \rho C_{p_g} u}{\partial \tau} = (\lambda_g u)'' - \rho v \frac{\partial C_{p_g} u}{\partial x} - \alpha_V (u - z_i) + \left(\Delta\alpha_V + u\frac{\partial \alpha_V}{\partial T_g}\right)(T_{s_i} - T_g), \tag{11}$$

supplemented by zero initial and boundary conditions

$$z_i(x, 0) = u(x, 0) = 0, \tag{12}$$

$$z_i(0, \tau) = u(0, \tau) = 0, \tag{13}$$

$$z_n(b, \tau) = u(b, \tau) = 0. \tag{14}$$

The following conditions are satisfied at the points between the layers

$$z_i(x_{i+1}, \tau) = z_{i+1}(x_{i+1}, \tau), \quad i = 1, 2, \ldots, n-1, \tag{15}$$

$$\frac{\partial z_i(x_{i+1}, \tau)}{\partial x} = \frac{\partial z_{i+1}(x_{i+1}, \tau)}{\partial x}. \tag{16}$$

For the linear part of the increment of functional (9), we have

$$\Delta J(\Delta\alpha_V, \Delta\lambda_s) = 2 \sum_{i=2}^{n} \int_0^{\tau_m} [T_s(x_i, \tau) - f_i(\tau)] \, z_i(x_i, \tau) \, d\tau. \tag{17}$$

For functional (9) to take extreme values, we need to equate to zero the first variation of the expanded functional, constructed similar to [8] and including the linear part of its increment, the initial and boundary conditions, and Eqs. (10) and (11).

Omitting the intermediate calculations and varying the independent variables (the condition of stationariness will be satisfied in the case of triviality of each group of terms with the corresponding variations [9]), we can write the conditions of the problem conjugate to problem (10)-(16):

$$-C_s \frac{\partial\psi_i}{\partial\tau} = \lambda_s \frac{\partial^2\psi_i}{\partial x^2} - \alpha_V(\psi_i - \varphi), \quad i = 1, 2, \ldots, n, \tag{18}$$

$$-\rho C_{p_g} \frac{\partial\varphi}{\partial\tau} = \lambda_g \frac{\partial^2\varphi}{\partial x^2} + \rho v C_{p_g} \frac{\partial\varphi}{\partial x} + (\psi_i - \varphi)\left(\alpha_V - \frac{\partial\alpha_V}{\partial T_g}(T_{s_i} - T_g)\right), \tag{19}$$

$$\psi_i(x, \tau_m) = \varphi(x, \tau_m) = 0, \tag{20}$$

$$\psi_i(0, \tau) = \varphi(0, \tau) = 0, \tag{21}$$

$$\psi_n(b, \tau) = \varphi(b, \tau) = 0, \tag{22}$$

$$\psi_i(x_{i+1}, \tau) = \psi_{i+1}(x_{i+1}, \tau), \quad i = 1, 2, \ldots, n-1, \tag{23}$$

$$\lambda_s\left(\frac{\partial\psi_i(x_{i+1}, \tau)}{\partial x} - \frac{\partial\psi_{i+1}(x_{i+1}, \tau)}{\partial x}\right) = 2[T_s(x_{i+1}, \tau) - f_{i+1}(\tau)]. \tag{24}$$

Considering Eqs. (24), (15), (13), and (14), for the linear part of the increment of the functional we have

$$\Delta J = \sum_{i=2}^{n}\int_0^{\tau_m} 2[T_s(x_i, \tau) - f_i(\tau)]\, z_i(x_i, \tau)\, d\tau = \sum_{i=2}^{n}\int_0^{\tau_m}\lambda_s\left[z_{i-1}(x_i, \tau)\frac{\partial\psi_{i-1}(x_i, \tau)}{\partial x} - z_i(x_i, \tau)\frac{\partial\psi_i(x_i, \tau)}{\partial x}\right] d\tau =$$

$$= \sum_{i=1}^{n}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\left[\lambda_s \frac{\partial z_i}{\partial x}\frac{\partial\psi_i}{\partial x} + z_i \frac{\partial}{\partial x}\left(\lambda_s \frac{\partial\psi_i}{\partial x}\right)\right] dx d\tau. \tag{25}$$

Using the conditions of the problem for the increments (10)-(16) and the problem conjugate to it (18)-(24) and integrating by parts, after transformations we obtain:

$$\sum_{i=1}^{n}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}} z_i \frac{\partial}{\partial x}\left(\lambda_s\frac{\partial\psi_i}{\partial x}\right) dx d\tau = -\sum_{i=1}^{n}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\lambda_s \frac{\partial z_i}{\partial x}\frac{\partial\psi_i}{\partial x} dx d\tau +$$

$$+\sum_{i=1}^{n}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\left\{u\psi_i\left(\alpha_V - \frac{\partial\alpha_V}{\partial T_g}(T_{s_i} - T_g)\right) - \Delta\alpha_V\,\psi_i(T_{s_i} - T_g) + \psi_i\frac{\partial}{\partial x}\left(\Delta\lambda_s\frac{\partial T_s}{\partial x}\right) - \alpha_V z_i\varphi\right\} dx d\tau.$$

Thus

$$\Delta J = \sum_{i=1}^{n}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\left\{u\psi_i\left(\alpha_V - \frac{\partial\alpha_V}{\partial T_g}(T_{s_i} - T_g)\right) - \Delta\alpha_V\,\psi_i(T_{s_i} - T_g) + \psi_i\frac{\partial}{\partial x}\left(\Delta\lambda_s\frac{\partial T_s}{\partial x}\right) - \alpha_V z_i\varphi\right\} dx d\tau. \tag{26}$$

Considering (20) and integrating by parts with allowance for conditions (12)-(16), we finally obtain

$$\Delta J = \sum_{i=1}^{n}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\left\{\psi_i\left[\frac{\partial^2\Delta\lambda_s}{\partial T_s^2}\left(\frac{\partial T_s}{\partial x}\right)^2 + \Delta\lambda_s\frac{\partial^2 T_s}{\partial x^2}\right] dx d\tau - \sum_{i=1}^{n}\int_0^{\tau_m}\int_{x_i}^{x_{i+1}}\Delta\alpha_V(T_{s_i} - T_g)(\psi_i - \varphi)\, dx d\tau. \tag{27}$$

We obtain the following for the gradient of the functional directly from Eq. (27)

$$\frac{\partial J}{\partial\alpha_V} = -(T_{s_i} - T_g)(\psi_i - \varphi). \tag{28}$$

The expression for the gradient of the functional $\partial J/\partial\lambda_s$ depends on the type of approximation of the relation $\lambda_s = \lambda_s(T_s)$. With the use of B-splines for this [10]:

$$\lambda_s(T_s) = \sum_{k=1}^{m+1} \lambda_k B_k(T_s)$$

the problem is reduced to finding the parameters λ_k, the gradient of the functional of which is determined by the expressions

$$\frac{\partial J}{\partial \lambda_k} = \sum_{i=1}^{n} \int_{x_i}^{x_{i+1}} \int_0^{\tau_m} \psi_i \left[\left(\frac{\partial T_s}{\partial x} \right)^2 \frac{\partial^2 B_k}{\partial T_s^2} + B_k(T_s) \frac{\partial^2 T_s}{\partial x^2} \right] dx d\tau.$$

To construct a stable algorithm based on the principle of variational regularization, we can choose the method of conjugate gradients. This is one of the most effective methods available from the point of view of the accuracy of the results, machine time, and amount of internal storage required.

The above algorithm was realized in a program written in FORTRAN and checked on a model sample involving establishment of the internal heat-transfer coefficient.

As the porous body we took a specimen of sintered copper powder with a porosity of 40% and of thickness b = 5 mm. It had the characteristics $\lambda_s = 0.1638 - 0.256 \cdot 10^{-4} T_s$, $\frac{\text{kW}}{\text{m} \cdot \text{K}}$, $C_s = 1.2155 - 0.984(T_s - 273) + 0.00234(T_s - 273)^2$, $\frac{\text{kJ}}{\text{m}^3 \cdot \text{K}}$. The chosen viscosity and inertial coefficients of resistance were as follows: $\alpha = 3.18 \cdot 10^{11}$, 1/m²; $\beta = 8.5 \times 10^6$, 1/m². The injected gas was air with a constant temperature at the inlet $T_g(0, \tau)$ = 300°K and injected at the rate ρv = 0.05 kg/m²·sec.

As the other boundary conditions, we took values obtained from solving the direct heat-transfer problem with the conditions

$$\alpha_V = 0.1 \frac{\rho v \lambda_g}{\mu_g d_p}, \quad \text{kW/m}^3 \cdot \text{K}; \quad q_w = 750 \left(\tau/\tau_m + \sin \frac{\pi \tau}{\tau_m} \right), \quad \text{kW/m}^2.$$

The internal heat-transfer coefficient is established from these boundary conditions.

The results obtained in solving the inverse problem after the fifth iteration were generalized by the formula (we used dimensional theory)

$$\alpha_V = 0.083 \left(\frac{\rho v}{\mu_g} \right)^{0.94} \frac{\lambda_g}{d_p^{1.06}},$$

which shows the good accuracy of the determination of the internal heat-transfer coefficient by the above approach.

NOTATION

x, coordinate; b, thickness of the porous body; n, number of measurements of the temperature of the body; C_s, λ_s, volumetric specific heat and thermal conductivity of the body; ρ, Cp_g, λ_g, density, specific heat, and thermal conductivity of the injected gas; T_s, T_g, temperature of the wall and gas; α_V, internal heat-transfer coefficient; ρv, blowing rate; τ, time; τ_m, duration of experiment; p, pressure; M_g, molecular weight of gas; α, β, coefficients of hydraulic resistance; ψ, φ, conjugate variables; q_w, heat flow to the wall at the boundary.

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FUZZY IDENTIFICATION OF ENERGY-EXCHANGE MODELS FROM TECHNOLOGICAL PROCESSES

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An identification method is proposed that enables one to estimate the parameters and also to evaluate the model performance. The method is based on fuzzy-set theory.

Very often, there are several competing models available to simulate a complex phenomenon or process, and these contain adapted parameters that require experimental identification.

Leaving aside aspects such as the computer run time and the algorithms involved, we can say that the best model is selected on criteria for accuracy and physical acceptability in the values obtained for the adapted parameters. The estimates of accuracy and physical acceptability are dependent on fuzzy factors involved in the subjective preferences of those developing and using the model, so it is desirable to use the theory of fuzzy sets to formalize the choice of the optimum model [1].

This analysis is made with reference to simulating the thermal and energy processes in the hot rolling of aluminum alloys. The following form can be given for the basic model for the processes in the rolling cage, which is represented by a system of nonlinear algebraic equations [2]:

$$T_1 = f(\sigma, \ \alpha, \ P, \ T_0, \ H_0, \ H_1, \ v), \tag{1}$$

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